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THREE-DIMENSIONAL RUNNING WAVES IN A BAROTROPIC GAS

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The equations of potential triple waves in a barotropic gas with an arbitrary equation of state are obtained. The properties of the solutions for contiguous flows of the double- and triple-wave type are investigated. The solutions of certain three- dimensional self-similar problems of three pistons are solved in the case of a "heavy" gas with a high initial velocity of sound. These problems concern three planes forming an infinite trihedral angle within which the gas is at rest at the instant $t = 0$, whereupon the planes begin to retract from the gas at high constant velocities.

1. A system of equations of triple waves for a polytropic gas in the hodograph space of the velocities u_1, u_2, u_3 was derived in [1]. Double waves in a barotropic gas for nonsteady potential two-dimensional flows were considered in [2] (see also Suchkov, Applying the method of differential constraints to gas dynamics problems. Candidate's thesis, Siberian Branch of the Academy of Sciences USSR, Novosibirsk). Some of the results of [2] constitute minor generalizations of the results obtained in [3,4] for a polytropic gas.

The equations of potential unsteady third-rank waves [1] for a gas with the equation of state $p = f(\rho)$ (p is the pressure, ρ is the density) can be derived exactly as for a polytropic gas. Proceeding as in [1], we introduce as our unknown functions the enthalpy

$$H(u_1, u_2, u_3) = \int \frac{dp}{\rho}$$

and the "deployment" function

$$\Pi(u_1, u_2, u_3) = \sum_{k=1}^{3} x_k u_k - \varphi - tH - \frac{t+1}{2}\,(u_1^2 + u_2^2 + u_3^2) \qquad (1.1)$$

Here x_k are Cartesian coordinates and φ is the velocity potential. We obtain the following system of equations for these functions H and Π :

$$\sum_{i,\,k=1}^{3} A_{ik}L_{ik}^{j}=0, \qquad j=1,2,3 \tag{1.2}$$

$$A_{ik}=\delta_{ik}-\frac{H_iH_k}{f'}, \quad H_k=\frac{\partial H}{\partial u_k}, \quad H_{mp}=\frac{\partial^2 H}{\partial u_m \partial u_p}$$

$$L_{ik}^{1}=(-1)^{i+k}\begin{vmatrix}\Pi_{mp}+\delta_{mp} & \Pi_{np}+\delta_{np}\\ \Pi_{mq}+\delta_{mq} & \Pi_{nq}+\delta_{nq}\end{vmatrix}, \quad \Pi_{mp}=\frac{\partial^2\Pi}{\partial u_m\partial u_p}$$

$$L_{ik}^{2}=(-1)^{i+k}\left\{\begin{vmatrix}\Pi_{mp}+\delta_{mp} & \Pi_{np}+\delta_{np}\\ H_{mq}+\delta_{mq} & H_{nq}+\delta_{nq}\end{vmatrix}+\begin{vmatrix}H_{mp}+\delta_{mp} & H_{np}+\delta_{np}\\ \Pi_{mq}+\delta_{mq} & \Pi_{nq}+\delta_{nq}\end{vmatrix}\right\}$$

$$L_{ik}^{3}=(-1)^{i+k}\begin{vmatrix}H_{mp}+\delta_{mp} & H_{np}+\delta_{np}\\ H_{mq}+\delta_{mq} & H_{nq}+\delta_{nq}\end{vmatrix} \qquad \begin{matrix}(m,\ n\neq k;\ m<n)\\ (p,\ q\neq i;\ p<q)\end{matrix}$$

Here δ_{mp} is the Kronecker delta.

Once the functions H and Π have been found, the flow in the physical space x_1, x_2, x_3, t can be determined from the formulas

$$x_i = u_i + \Pi_i + t\,(u_i + H_i), \qquad i = 1,2,3 \tag{1.3}$$

Equations (1.2), (1.3) were obtained under the assumption that u_1, u_2, u_3 are functionally independent.

System (1.2) constitutes an overdetermined system of three equations for two unknown functions; the equation for H is independent. The second partial derivatives of the functions H and Π occur quadratically in all of the equations. After finding H we must choose a Π which satisfies Eqs. (1.2) for $j = 1,2$. For $\Pi = -{}^1/_2\,(u_1^2 + u_2^2 + u_3^2)$ these equations are fulfilled automatically, and the equation for H describes self-similar flows which depend on the variables x_i / t, $i = 1,2,3$. Sample nonself-similar flows of the triple-wave type are constructed in [5].

If u_1, u_2, u_3 are functionally dependent and if the flow corresponds to the surface $u_3 = \psi\,(u_1, u_2)$ in the hodograph space, then (as in [1]) it is easy to obtain an already closed system of three equations for the functions Ψ, H and Π (second-rank waves) in the plane u_1, u_2 in the form

$$L_1(\Psi) = R_{11}\Psi_{22} - 2R_{12}\Psi_{12} + R_{22}\Psi_{11} = 0 \tag{1.4}$$

$$L_2(H) = R_{11}(H_{22} + 1 + \Psi_2^2) - 2R_{12}(H_{12} + \Psi_1\Psi_2) + R_{22}(H_{11} + 1 + \Psi_1^2)=0$$

$$L_3(\Pi) = R_{11}(\Pi_{22} + 1 + \Psi_2^2) - 2R_{12}(\Pi_{12} + \Psi_1\Psi_2) + R_{22}(\Pi_{11} + 1 + \Psi_1^2) = 0$$

where

$$R_{ik}=\delta_{ik}+\Psi_i\Psi_k-\frac{1}{f'}H_iH_k, \quad \Psi_{ik}=\frac{\partial^2\Psi}{\partial u_i\partial u_k}, \quad \Psi_i=\frac{\partial\Psi}{\partial u_i}$$

The flow in physical space can be reconstructed from the formulas

$$\Pi_i + u_i + \Psi\Psi_i + (H_i + u_i + \Psi\Psi_i)\,t = x_i + x_3\Psi_i, \qquad i = 1, 2 \tag{1.5}$$

This system of equations for a polytropic gas was first obtained in [6].

Let us consider the properties of flows with contiguous second- and third-rank waves. These will be found useful in the solution of specific problems. The case of contiguous first-rank (simple) and second-rank waves was investigated in [7].

Property 1. If the surface $u_3 = \Psi(u_1, u_2)$ corresponds in physical space to the surface $F(x_1, x_2, x_3, t) = 0$ along which some triple wave is contiguous to a three-dimensional double wave, then $u_3 = \Psi(u_1, u_2)$ is the characteristic surface for system (1.2).

Let the initial conditions

$$H = H(u_1, u_2), \quad H_\psi = H_\psi^*(u_1, u_2), \quad \Pi = \Pi(u_1, u_2), \quad \Pi_\psi = \Pi_\psi(u_1, u_2) \quad (1.6)$$

be given at the surface $u_3 = \Psi$.

Here H_ψ and Π_ψ are the generating derivatives.

Replacing u_3 in Eqs. (1.2) by the new independent variable $\lambda = u_3 - \Psi(u_1, u_2)$, we can rewrite system (1.2) as

$$AH_{\lambda\lambda} + C_1 = 0, \qquad AH_{\lambda\lambda} + B\Pi_{\lambda\lambda} + C_2 = 0 \qquad B\Pi_{\lambda\lambda} + C_3 = 0$$
$$\Pi_{\lambda\lambda} = \frac{\partial^2 \Pi}{\partial \lambda^2}, \quad H_{\lambda\lambda} = \frac{\partial^2 H}{\partial \lambda^2} \quad (1.7)$$

$$A = L_2(H) - (H_\psi - \Psi) L_1(\Psi), \quad B = L_2(\Pi) - (H_\psi - \Psi) L_1(\Psi) \quad (1.8)$$

Here the coefficients of $H_{\lambda\lambda}$ and $\Pi_{\lambda\lambda}$ vanish, and the coefficients C do not depend on $H_{\lambda\lambda}$, $\Pi_{\lambda\lambda}$. The surface $u_3 = \Psi$ is characteristic only if $A = B = 0$. The latter equations follow in this case from (1.4) by virtue of (1.8). Property 1 has been proved.

Remark. In the case where a self-similar triple wave is contiguous to a two-dimensional double wave with $\Psi = \text{const}$ the equation of the characteristic surface for the triple wave equation simply coincides with the double-wave equation.

Now let us consider the problem of determining the shape of the contiguity surface in physical space in the case of self-similar flows.

Let the function $H(u_1, u_2)$ for a double self-similar wave $(\Pi = -{}^1/_2\,(u_1^2 + u_2^2))$ be known and let $\Psi \equiv 0$.

Property 2. The shape of the contiguity surface $F(\xi_1, \xi_2, \xi_3) = 0$, $\xi_i = x_i / t$ of the double- and triple-wave domains can be determined without solving triple-wave equation (1.2) for $j = 3$, as the required result can be obtained by solving a certain first-order partial differential equation.

In fact, the contiguity surface in the space ξ_1, ξ_2, ξ_3 can be found in this case by eliminating u_1, u_2 from the relations

$$\xi_i = u_i + H_i(u_1, u_2), \; i = 1, 2; \; \xi_3 = H_3(u_1, u_2) \quad (1.9)$$

Let $H(u_1, u_2)$ on the plane $u_3 = 0$ correspond to the double wave, so that $H_1, H_2, H_{11}, H_{12}, H_{22}$ as functions of u_1, u_2 are known. The contiguity surface will be determined once we know $H_3(u_1, u_2) = \Phi(u_1, u_2)$. To find the function Φ from system (1.2) for $j = 3$ we set $H_{31} = \Phi_1$, $H_{32} = \Phi_2$ $(\partial\Phi / \partial u_i = \Phi_i)$ and recall the fact that the coefficient H_{33} vanishes. This gives us the first-order partial differential equation

$$\left(1 - \frac{H_1^2}{f'}\right)\Phi_2^2 + \left(1 - \frac{H_2^2}{f'}\right)\Phi_1^2 + \left(\frac{\Phi^2}{f'} - 1\right)(H_{11}H_{22} + H_{11} + H_{22} - H_{12}^2 + 1) +$$
$$+ \frac{H_1 H_2}{} \Phi_1\Phi_2 + \frac{2}{f'}(H_1 H_{12} - H_2 H_{11} - H_2)\Phi\Phi_2 + \frac{2}{f'}(H_2 H_{12} - H_1 H_{22} - H_1)\Phi\Phi_1 = 0 \quad (1.10)$$

In specific problems the function Φ is usually known on some line $\varphi(u_1, u_2) = 0$, or from the conditions of contiguity with double waves, or from the conditions at movable walls [5]. Thus, by solving (1.10) with the given initial conditions by ordinary methods (e.g. by reducing the problem to a system of ordinary differential equations for the characteristics) to find $\Phi(u_1, u_2)$, it is possible to find the shape of the contiguity surface directly.

Let us consider the special case of contiguity with a double wave of special form in a polytropic gas,

$$c = \alpha_0 + \alpha_1 u_1 + \alpha_2 u_2 \tag{1.11}$$

(c is the velocity of sound; $\alpha_0 = \text{const}$, $\alpha_1 = 0.5(\gamma - 1)$, $\alpha_2 = 0.5(\gamma - 1)(1 + \gamma)^{1/2}(3 - \gamma)^{-1/2}$; γ is the adiabatic exponent). Such double waves were investigated in [5,8]. They are important in the solution of several problems concerning the retraction of plane pistons from a gas. Equation (1.10) can be simplified in this case. Replacing H by the function $c(u_1, u_2, u_3)$ ($c^2 = dp / d\rho$), setting $\partial c / \partial u_3 = \Gamma(u_1 u_2)$ for $u_3 = 0$, and substituting variables according to the expressions

$$\xi = \alpha_0 + \alpha_1 u_1 + \alpha_2 u_2, \qquad \eta = -\alpha_2 u_1 + \alpha_1 u_2$$

we obtain the following equation for Γ :

$$4\frac{\gamma - 2}{(\gamma - 1)^2}\Gamma^2 + 8\frac{\xi}{\gamma - 1}\Gamma\Gamma_\xi - 4\frac{\xi^2}{3 - \gamma}\Gamma_\xi^2 + 4\frac{\xi^2(\gamma + 1)}{(3 - \gamma)^2}\Gamma_\eta^2 + \frac{\gamma + 1}{3 - \gamma} = 0 \tag{1.12}$$

In the particular problem of escape into a vacuum along a dihedral angle (see [5]) we have the condition

$$\Gamma = \alpha_3 = \frac{\gamma - 1}{2}\left(\frac{\gamma + 1}{(3 - \gamma)(2 - \gamma)}\right)^{1/2} = \text{const} \quad \text{for } u_3 = 0$$

Equation (1.12) is fulfilled ($\Gamma_\xi = \Gamma_\eta = 0$) for $\Gamma \equiv \alpha_3 = \text{const}$. Replacing H by the function $c(u_1, u_2, u_3)$ in (1.9), we obtain the following equation for the contiguity surface (in this case a plane):

$$\alpha_3 + \alpha_1\alpha_3\xi_1 + \alpha_2\alpha_3\xi_2 - (\alpha_1 + \alpha_1^2 + \alpha_2^2)\xi_3 = 0 \tag{1.13}$$

2. It would be of interest to obtain the exact particular solutions of (1.2) appearing in [5] or to construct approximate solutions of certain problems. We shall construct approximate solutions of the equation for H (the self-similar case) and pictures of motion in physical space for the case of interaction of three-dimensional rarefaction waves in a "heavy" gas.

Let a homogenous gas with a high velocity of sound c ($c \gg 1$; m/sec can be taken as the units of measurement) be at rest inside an infinite trihedral angle bounded by the planes P_1, P_2, P_3; let the dihedral angles between these planes be $\alpha_{12} \leqslant \frac{1}{2}\pi$, $\alpha_{13} \leqslant \frac{1}{2}\pi$, $\alpha_{23} \leqslant \frac{1}{2}\pi$. At the instant $t = 0$ the planes P_i (pistons) begin to retract from the gas with low (as compared with the velocity of sound in the unperturbed gas) constant velocities V_i. The resulting self-similar flow depends on the variables $\xi_i = x_i / t$, $i = 1, 2, 3$.

We note that the acoustic approximation is generally inadequate for this problem, since the gradients of the required quantities are large.

Let us consider the class of flows in which strong discontinuities do not arise. Such flows are potential and consist of domains of constant motion of simple, double, and triple waves (the triple wave is described by Eq. (1.2) for $j = 3$). In [7,8] we considered two-dimensional problems on the retraction from a polytropic "heavy" gas of two pistons P_1, P_2 with the angle α between them at the low velocities V_1, V_2. We showed that the complete potential flow can be constructed only for $\alpha = \pi / k$ (k is an integer).

Let us replace H by $M = \int cp^{-1}dp$ ($M = (2/\gamma - 1)c$ in a polytropic gas) as our unknown function.

The equation for M which follows from (1.2) for $j = 3$ can be reduced to the form

$$\sum_{i,k=1}^{3} (\delta_{ik} - M_i M_k) \begin{vmatrix} M_{mp} + aM_m M_p + b\delta_{mp} & M_{np} + aM_n M_p + b\delta_{np} \\ M_{mq} + aM_m M_q + b\delta_{mq} & M_{nq} + aM_n M_q + b\delta_{nq} \end{vmatrix} = 0 \qquad (2.1)$$

where

$$m, n \neq k, \quad m < n; \qquad p, q \neq i, \quad p < q$$

$$a = \frac{1}{2}\frac{f''p}{f'^{3/2}}, \qquad b = \frac{1}{f'^{1/2}}, \qquad M_{np} = \frac{\partial^2 M}{\partial u_n \partial u_p}$$

Relations (1.3) become

$$\xi_i = u_i + \sqrt{f'} M_i \qquad (2.2)$$

Weak discontinuities begin to move with the velocity $c = \sqrt{f'}$ into the unperturbed gas from the planes P_i. Since the dimensions of the flow region in the space ξ_1, ξ_2, ξ_3 are of the order $O(c)$, it follows from formulas (2.2) that the quantities M_i are bounded and that $M_i = O(1)$. Let us assume, moreover, that the inequality $a \ll 1$ is fulfilled throughout the flow region. This condition is fulfilled when $c \gg 1$ for ordinary equations of state, including that of a polytropic gas.

The above assumptions enable us to neglect terms containing a and b ($b \ll 1$) as factors in (2.1). The resulting approximate equation for M admits of a solution of the form

$$M = d_0 + d_1 u_1 + d_2 u_2 + d_3 u_3 \qquad (2.3)$$

where d_i are arbitrary constants. Let us use these solutions to construct flows in the three-piston problem. We shall also make use of linear relations between M and the components u_i in the simple- and double-wave regions. Such relations are exact for simple waves (they are obtainable from the exact equations of hydrodynamics); in the case of double waves they are obtained in the same way as for a triple wave. It is easy to see that the conclusions of [8] (the case of a heavy gas) are also valid for an arbitrary equation of state; the solutions obtained in [8] in the hodograph plane for the function $(2/\gamma - 1)c$ are of the same form for the new unknown function M. Hence, in order to construct the complete flow without strong discontinuities we must assume that the dihedral angles $\alpha_{12}, \alpha_{13}, \alpha_{23}$ are of the form

$$\alpha_{12} = \pi / k_1, \qquad \alpha_{13} = \pi / k_2, \qquad \alpha_{23} = \pi / k_3 \qquad (2.4)$$

where $k_i \geq 2$ are integers. This follows from the fact that planar flows far away from the vertex of the trihedral angle formed by the moving pistons are adjacent to the ribs.

But for an arbitrary trihedral angle which does not degenerate into a ray we have

$$\alpha_{12} + \alpha_{13} + \alpha_{23} > \pi \qquad (2.5)$$

This inequality and (2.4) imply that we need only consider the following possible combinations of the angles $\alpha_{12}, \alpha_{13}, \alpha_{23}$:

$$\left(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{k}\right) \ (k \geqslant 2 - \text{ is an integer}) \ \left(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{3}\right), \ \left(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}\right), \ \left(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{5}\right)$$

Leaving aside the three exceptional cases, let us consider the principal possiblity $(^1/_2\pi, {^1/_2}\pi, \pi / k)$.

The equations of the angle faces at the initial instant can be specified in the form

$$x_1 = 0, \qquad x_3 = 0, \qquad x_1 = x_2 \operatorname{ctg}(\pi / k) \tag{2.6}$$

In the hodograph space the flow domains correspond to the prism bounded by the planes

$$u_3 = 0, \quad u_2 = 0, \quad u_1 = -V, \quad u_1 = -u_2 \operatorname{ctg}(\pi / k) \tag{2.7}$$
$$u_3 = -V, \quad -u_1 \cos(\pi / k) + u_2 \sin(\pi / k) + V = 0$$

The top base of the prism (the plane $u_3 = 0$) is divided by straight lines into areas corresponding to the various planar double and simple waves as in [•]. The triple-wave regions can be obtained by dividing the prism into segments by planes parallel to the axis u_3 and passing through the above straight lines in the plane $u_3 = 0$.

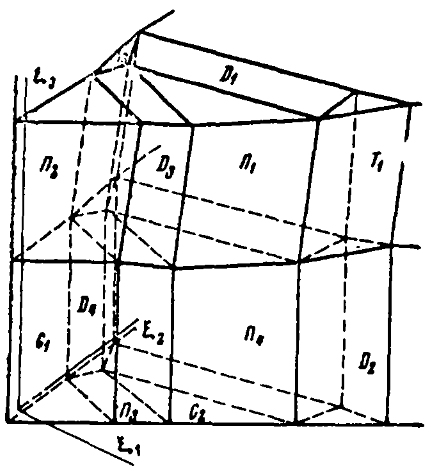

Fig. 1

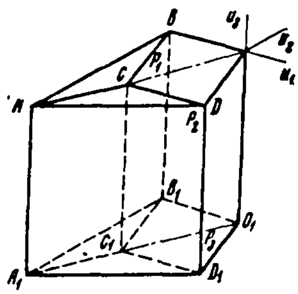

Fig. 2

The flow is defined in physical space by formulas (2.2). For example, let us consider the motion of a polytropic gas for the trihedral angle $(^1/_2\pi, {^1/_2}\pi, {^1/_3}\pi)$ $(k = 3)$. Figure 1 shows (not in scale) the flow region in the hodograph space when the pistons P_i retract with equal velocities V. The plane AOO_1A_1 is a plane of symmetry in this case; it can be regarded as a stationary wall.

Figure 2 shows (not in scale) the part of the flow region in physical space which is bounded by the pistons P_1, P_2 and the stationary wall (the bisector plane). The regions denoted by the letters T_i $(i = 1,2)$ correspond to the triple-wave regions; D_j $(j = 1, 2, 3, 4)$ to the double-wave regions; Π_k $(k = 1,2,3,4)$ to the simple-wave regions; C_l $(l = 1,2)$ to the constant flow regions.

Table 1.

Space ξ_1, ξ_2, ξ_3	Space u_1, u_2, u_3	Solution in the hodograph space
	Prisms	
T_1	$BCDOO_1D_1C_1B_1$	$M = u_1 + \sqrt{3} u_2 + u_3 + M_0$
T_2	$ABCC_1B_1A_1$	$M = {}^2/_3 \sqrt{3} u_2 + u_3 + M_0 - V$
	Planes	
D_1	BCC_1B_1	$M = u_3 + {}^2/_3 \sqrt{3} u_2 + M_0 - V, \quad u_2 = -\sqrt{3}(u_1 + V)$
D_2	$B_1C_1D_1O_1$	$M = u_1 + \sqrt{3} u_2 + M_0 - V, \quad u_3 = -V$
D_3	ACC_1A_1	$M = {}^2/_3 \sqrt{3} u_2 + u_3 + M_0 - V, \quad u_2 = \sqrt{3} u_1$
D_4	$A_1B_1C_1$	$M = {}^2/_3 \sqrt{3} u_2 + M_0 - 2V, \quad u_3 = -V$
	Straight lines	
Π_1	CC_1	$u_1 = -{}^1/_2 V, \quad u_2 = -{}^1/_2 \sqrt{3} V, \quad u_3 = M - M_0 + 2V$
Π_2	AA_1	$u_1 = -V, \quad u_2 = -\sqrt{3} V, \quad u_3 = M - M_0 + 3V$
Π_3	A_1C_1	$u_1 = {}^1/_2 (M - M_0 + 2V), \quad u_2 = {}^1/_2 \sqrt{3} (M - M_0 + 2V)$ $u_3 = -V$
Π_4	B_1C_1	$u_1 = -{}^1/_2 (M - M_0 + 4V), \quad u_2 = {}^1/_2 \sqrt{3} (M - M_0 + 2V)$ $u_3 = -V$
	Points	
C_1	A_1	$u_1 = -V, \quad u_2 = -\sqrt{3} V, \quad u_3 = -V, \quad M = M_0 - 4V$
C_2	C_1	$u_1 = -{}^1/_2 V, \quad u_2 = -{}^1/_2 \sqrt{3} V, \quad u_3 = -V, \quad M = M_0 - 3V$

The above letters are placed at one of the faces of the corresponding region. Table 1 contains the solution in the hodograph space for the function $M = 2c/(\gamma - 1)$; here M_0 is the value of M in the unperturbed gas. Formulas (2.2) in this case become

$$\xi_i = u_i + {}^1/_2 (\gamma - 1) M M_i$$

The triple wave T_1 in the space ξ_1, ξ_2, ξ_3 is bounded by the planes

$${}^1/_2 (\gamma - 1)(\xi_1 + \sqrt{3}\xi_2 + M_0) - (2\gamma - 1)\xi_3 = 0$$

$$\xi_1 + {}^1/_3 \sqrt{3}\, \xi_2 + V + (\gamma - 1)({}^3/_2 \xi_1 - {}^1/_6 \sqrt{3} \xi_2 - \xi_3 - M_0 + {}^5/_2 V) = 0$$

$${}^1/_2 (\gamma - 1)(\xi_1 + \xi_3 + M_0) - \gamma \xi_2 = 0$$

$$\frac{\gamma - 1}{2}(\xi_1 + \sqrt{3}\xi_2 + M_0) - (2\gamma - 1)\xi_3 + \frac{\gamma - 3}{2} V = 0$$

$$\xi_2 - \sqrt{3}\xi_1 = 0 \qquad \text{(stationary wall)}$$

The triple wave T_2 is bounded by the planes

$$\tfrac{1}{2}\sqrt{3}(\gamma-1)\,\xi_2-\tfrac{1}{2}(\gamma+1)\,\xi_3+\tfrac{1}{2}(\gamma-1)(M_0-V)=0$$
$$\tfrac{1}{2}(7\gamma-1)\,\xi_1+\tfrac{1}{2}\sqrt{3}(\gamma+1)\,\xi_2-(\gamma-1)(\xi_3+M_0)+\tfrac{3}{2}(3\gamma-1)V=0$$
$$\tfrac{1}{2}(7\gamma-1)\,\xi_1-\tfrac{1}{2}\sqrt{3}(\gamma+1)\,\xi_2+(\gamma-1)(\xi_3+M_0-V)=0$$
$$\tfrac{1}{2}\sqrt{3}(\gamma-1)\,\xi_2-\tfrac{1}{2}(\gamma+1)\,\xi_3+\tfrac{1}{2}(\gamma-1)(M_0-\tfrac{10}{3}V)=0$$
$$\xi_1=-V\qquad\text{(piston)}$$

All of the side faces of the regions in the lower half of Fig. 2 are orthogonal to the piston $\xi_3=-V$.

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MOTION OF A HEAT-CONDUCTING GAS ACTED ON BY A HEAT-INSULATED EXPANDING PISTON

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The temperature and density fields associated with the motion of an ideal gas acted on by an expanding piston have singularities at the piston surface [1-3]. These arise through nonallowance for heat conduction by the gas, which plays the determining role near the surface of the piston.

We shall solve the problem of motion of a heat-conducting gas acted on by an expanding heat-insulated piston by the method of interior and exterior expansions. To this end we construct the principal term of the interior asymptotic expansion by splicing it with the solution for an ideal gas which constitutes the principal term of the exterior